where C_1 and C_2 are arbitrary constants. We note that when $\alpha = 0$ (8) changes into the well-known Nadai solution [3], and when $\alpha \neq 0$ we obtain a new velocity field.

Remark. The condition that the energy dissipation be positive imposes the following constraint on the value of the parameters: $\beta - \alpha y > 0$ if $y \in [-h, h]$.

We shall construct the possible invariant solutions on the subgroup $X_1 + \alpha X_5$. We shall seek a solution invariant with respect to this subgroup in the form

$$u = f(y)e^{\alpha x},\ v = g(y)e^{\alpha x}. \tag{9}$$

Substituting (9) into (6), we obtain a system of ordinary differential equations

$$y(\alpha f - g') = (f' + \alpha g)\sqrt{h^2 - y^2},\ \alpha f + g' = 0.$$

From this we have

$$\sqrt{h^2 - y^2}g'' + 2\alpha^2 g' y - \alpha^2\sqrt{h^2 - y^2}g = 0.$$

The latter equation reduces by the substitution $g' = gu$ to the Riccati equation

$$u' + u^2 + 2\alpha^2 \frac{y}{\sqrt{h^2 - y^2}} u + \alpha^2 = 0.$$

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STABILITY OF A VISCOPLASTIC RING

S. V. Serikov

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We present a theoretical study of the unsteady deformation of cylindrical metal shells under impulsive loading. We investigate the stability of the inertial motion of the boundaries of a flat ring toward or away from the center under small harmonic perturbations of the boundaries, the velocity, and the stress tensor. We derive a relation for the wave number at which the motion becomes unstable, and compare the result with experimental data.

1. Examples of Modeling Processes. In contrast with articles on the dynamic buckling of cylindrical shells under impulsive external or internal pressures [1-5], we consider problems with large plastic deformations (of the order of 100%). Our method of treating the mechanism of the development of unstable motion is similar to that employed in papers on the instability of motion of a finite mass of liquid with a free boundary [6-8].

Figure 1 shows the result of an experiment on the axisymmetric compression of a D16 Duralumin cylindrical shell by detonation products. The initial outside diameter, wall thickness, and height of the shell were, respectively, 22 × 2.5 × 80 mm. After the experiment the average dimensions were 9.4 × 3.9 × 80 mm with an internal square opening (Fig. 1, magnification 10 ×). In the drawing of a 10 × 2 mm 12Kh1MF steel tube to 6 × 2.2 mm without a mandrel, a square channel is formed (Fig. 2, magnification 10 ×). If we consider another method of longitudinal milling of seamless tubing, namely the reduction of 86 × 10 mm 20 St tubing without a mandrel to 65 × 11 mm in two-roller circular-oval passes, we obtain a square internal channel (Fig. 3).

These examples show that over a wide range of initial deformation parameters of tubes (velocity of boundaries 1-1000 m/sec, mechanical properties of the shell material, etc.) we have a characteristic internal profile. In a number of cases in the drawing and reduction of thick-walled tubes, hexagonal, octagonal, etc. internal channels are formed [9, 10]. Wavy boundaries are formed in the hot drawing of seamless tubes (Fig. 4). Here it is believed

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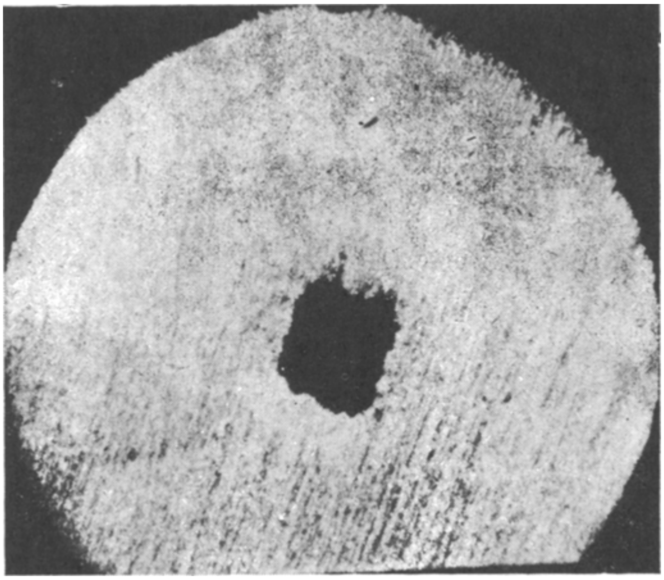

Fig. 1

that in the unsteady deformation of a cylindrical shell toward its axis the inner free boundary becomes wavy with a certain predominant wavelength as a result of unstable motion under small perturbations of the boundaries and velocity.

We now consider the motion of a ring away from its axis. The shell under intense dynamic loading of its inner boundary expands, and may fracture into individual pieces [11-13]. Here the assumption of the correlation of the number of fragments formed and the formation of unstable harmonics on the shell boundaries has a simple physical meaning.

By making a certain approximation we investigate the stability of the inertial motion of a circular shell of viscoplastic material without restrictions on the initial wall thickness and diameter.

2. Formulation of the Problem. We consider unsteady plane strain without twisting of an incompressible viscoplastic ring. Let r and θ be plane polar coordinates with the origin at the center of the ring, and $t \geqslant 0$ the time. The components of the stress tensor σ_r, σ_θ, $\sigma_{r\theta}$, and the components of the velocity vector v_r and v_θ under unsteady strain of a medium in a closed region with a variable boundary are determined from the following equations.

The equations of motion of a continuous medium outside the field of external body forces are

$$\begin{gathered}\frac{\partial \sigma_r}{\partial r}+\frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta}+\frac{\sigma_r-\sigma_\theta}{r}=\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{1}{r}v_\theta\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}\right),\\ \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta}+\frac{\partial \sigma_{r\theta}}{\partial r}+\frac{2\sigma_{r\theta}}{r}=\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{1}{r}v_\theta\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}\right),\end{gathered} \tag{2.1}$$

where ρ is the density of the medium.

The assumption of incompressibility of the medium leads to the familiar relation

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}=0. \tag{2.2}$$

In the two-dimensional case an incompressible viscoplastic material is described by the equations [14]

$$\begin{gathered}\sigma_r=\sigma+2\mu\frac{\partial v_r}{\partial r}-\frac{\sigma_s}{2}\cos 2\theta_1,\quad \sigma=\frac{1}{2}(\sigma_r+\sigma_\theta),\\ \sigma_\theta=\sigma+2\mu\left(\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right)+\frac{\sigma_s}{2}\cos 2\theta_1,\\ \sigma_{r\theta}=\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta}+\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}\right)-\frac{\sigma_s}{2}\sin 2\theta_1,\\ \operatorname{tg} 2\theta_1=\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta}+\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}\right)\Big/\left(\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{\partial v_r}{\partial r}\right),\end{gathered} \tag{2.3}$$

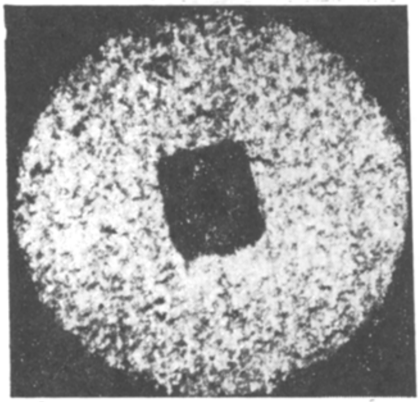

Fig. 2

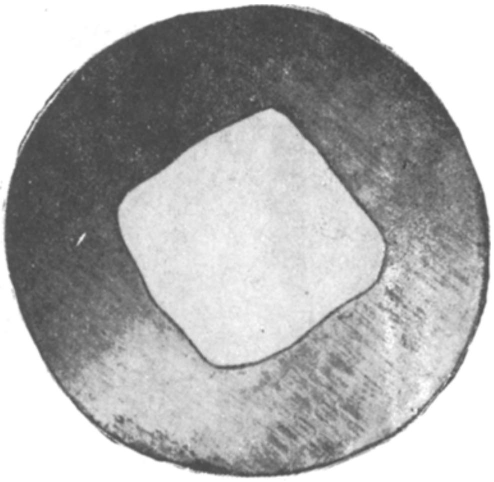

Fig. 3

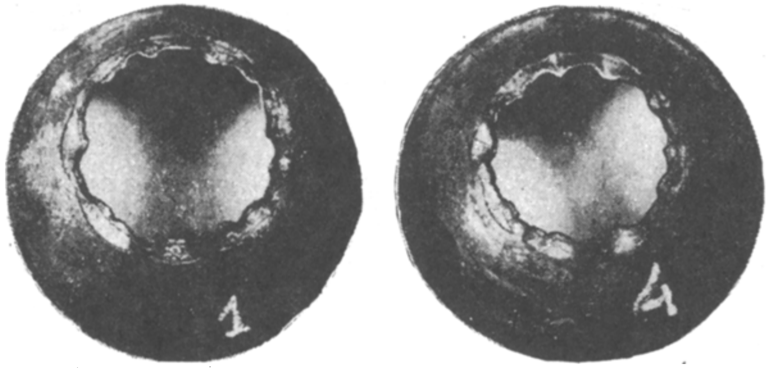

Fig. 4

where $\sigma_s \geqslant 0$ is the dynamic yield point, and $\mu \geqslant 0$ is the dynamic viscosity. For plane strain we take the axial component of the stress tensor $\sigma_z = \sigma$.

Let $F_i(t, r, \theta) = 0$ be the equations of the boundaries of the annular region which expands or contracts toward its center. We require that the kinematic condition

$$\frac{\partial F_i}{\partial t} + v_r \frac{\partial F_i}{\partial r} + \frac{1}{r} v_\theta \frac{\partial F_i}{\partial \theta} = 0 \quad (i = 1, 2) \tag{2.4}$$

and the dynamic condition

$$\sigma_r \cos(r, n) + \sigma_{r\theta} \cos(\theta, n) = 0, \ \sigma_{r\theta} \cos(r, n) + \sigma_\theta \cos(\theta, n) = 0, \tag{2.5}$$

be satisfied on the load-free boundaries, where the direction cosines of the outward normal n to the boundaries are with respect to the coordinate axes in the plane.

At time t = 0 we specify the region occupied by the incompressible viscoplastic medium with its boundaries, and the initial velocity distribution.

Equations (2.1)-(2.5) together with the initial data form a closed mathematical model of the unsteady strain of a viscoplastic material in a region with a variable boundary. For $\sigma_s = 0$ and $\mu \neq 0$ we have the case of an incompressible viscous liquid, for $\sigma_s \neq 0$ and $\mu = 0$, the model of ideal plasticity, and for $\sigma_s = 0$ and $\mu = 0$ Eqs. (2.1)-(2.5) describe an ideal incompressible liquid [6, 7].

3. Fundamental Motion of a Ring. We consider two cases of loading of a ring formed by two concentric circles of radii R_1 and R_2 ($R_1 < R_2$). In the first case the viscoplastic ring converges axisymmetrically toward its center for a given initial velocity distribution. In the second case there is axisymmetric inertial expansion of the ring.

The exact solution of Eqs. (2.1)-(2.5) for axisymmetric motion of a viscoplastic ring is known [15]. Before writing the solution for the cases under consideration, we introduce the dimensionless quantities

$$\bar{r} = r/R_{10},\quad \bar{t} = tV_{10}/R_{10},\quad \bar{\sigma}_r = \sigma_r/\rho V_{10}^2,\quad \bar{\sigma}_\theta = \sigma_\theta/\rho V_{10}^2,$$
$$\bar{\sigma}_{r\theta} = \sigma_{r\theta}/\rho V_{10}^2,\quad \bar{v}_r = v_r/V_{10},\quad \bar{v}_\theta = v_\theta/V_{10},\quad \bar{R}_i = R_i/R_{10}, \tag{3.1}$$
$$\sigma_* = \sigma_s/\rho V_{10}^2,\quad \nu = \mu/\rho V_{10}R_{10},\quad \varkappa_0 = R_{20}/R_{10},\quad V_{10} \neq 0,$$

and from now on for simplicity we omit the bars over the dimensionless quantities.

For axisymmetric motion of a ring we obtain from (2.1)-(2.5) and (3.1) the following relations for the components of the stress tensor:

$$\sigma_{r,j} = (\ddot{R}_j R_j + \dot{R}_j^2 + \sigma_*) \ln r/R_j + \frac{1}{2}\dot{R}_j R_j (4\nu - \dot{R}_j R_j)(R_j^{-2} - r^{-2}), \tag{3.2}$$
$$\sigma_{\theta,j} = \sigma_{r,j} + \sigma_* + 4\nu \dot{R}_j R_j r^{-2},\ \sigma_{r\theta,j} = 0,$$

where j = 1 corresponds to the solution for the converging ring, and j = 2 for the diverging ring. A dot over a quantity denotes its time derivative, and in both cases the components of the velocity vector are

$$v_r = \dot{R}_1 R_1 r^{-1},\ v_\theta = 0,\ \dot{R}_1 R_1 = \dot{R}_2 R_2, \tag{3.3}$$

and the equation of the boundaries of the ring is given by $F_i = r - R_i$.

The law of variation of the inner boundary of the viscoplastic ring is determined from the Cauchy problem for the second-order nonlinear condition

$$R_1\ddot{R}_1 + a_1\dot{R}_1^2 + a_2\dot{R}_1 + \sigma_* = 0,\ R_1 = 1,\ \dot{R}_1 = \mp 1 \ \text{ at } \ t = 0,$$
$$a_1 = 1 + \frac{(\varkappa_0^2 - 1)}{wR_1^2 \ln w},\ w = 1 + (\varkappa_0^2 - 1)/R_1^2,\ a_2 = 4\nu(\varkappa_0^2 - 1)/wR_1^3 \ln w. \tag{3.4}$$

For inertial convergence of the ring toward the center we take $\dot{R}_1(0) = -1$ in the initial data, and for divergence of the ring $R_1(0) = 1$. Since the ring material is incompressible, its outside radius is given by

$$R_2 = (R_1^2 + \varkappa_0^2 - 1)^{1/2}.$$

The ordinary second-order differential equation (3.4) is reduced to a first-order equation by the substitutions

$$z = \dot{R}_1 R_1,\ \dot{R}_1 = z/R_1,\ \ddot{R}_1 = (z'R_1 - z)zR_1^{-3}, \tag{3.5}$$

and then we obtain from (3.4) an Abel differential equation of second kind [16] in the form

$$zz' + (a_1 - 1)R_1^{-1}z^2 + a_2 z + \sigma_* R_1 = 0,\quad z(1) = \mp 1. \tag{3.6}$$

Here and later a prime denotes differentiation with respect to R_1. Suppose at t = 0 the ring is thin-walled with $\varkappa_0 = 1 + \varepsilon_0$, and $\varepsilon_0 = s_0/R_{10} \ll 1$, where s_0 is the initial thickness of the ring wall. From (3.4) and (3.6) we obtain

$$R_1 zz' - z^2 + 4\nu z + R_1^2\sigma_* = 0,\quad z(1) = \mp 1, \tag{3.7}$$

which is accurate to terms of first order in ε_0.

By introducing the new function $u = (z - 4\nu)/R_1$, and substituting it into (3.7), we obtain $u'(4\nu + R_1 u) = -\sigma_*$. Taking $u = u(t)$ as the unknown variable, we reduce the last nonlinear equation to a linear equation in R_1. The solution of (3.7) for $\sigma_* \neq 0$ has the form

$$R_1 = e^{-f(u)}\left(1 - \frac{4\nu}{\sigma_*}\int_{\mp 1 - 4\nu}^{u} e^{f(\tau)}\,d\tau\right),\quad f(u) = [u^2 - (\mp 1 - 4\nu)^2]/2\sigma_*. \tag{3.8}$$

For $\sigma_* = 0$, $\nu \neq 0$, and $\sigma_* \neq 0$, $\nu = 0$, we obtain from (3.7), respectively,

$$z = \mp R_1 + 4\nu(1 - R_1),\quad z = \mp R_1\sqrt{1 - 2\sigma_* \ln R_1}. \tag{3.9}$$

The asymptotic solution of (3.7) is needed later for the investigation of the stability of axisymmetric motion of a viscoplastic ring under small perturbations of the boundaries. An analysis of (3.7)-(3.9) shows that the asymptotic solutions of (3.7) for $\sigma_* \neq 0$, $\nu \neq 0$ for convergence and divergence of the ring are, respectively,

$$z \sim 4\nu \ \text{ as } \ R_1 \to 0,$$

$$z \to R_1\sqrt{1-2\sigma_* \ln R_1} \quad \text{as} \quad R_1 \to R_{1*} \quad (R_{1*} = e^{1/2\sigma_*}). \tag{3.10}$$

Thus, from (3.3)-(3.10) we obtain the asymptotic representations of the fundamental motion:

for the converging ring ($R_1 \to 0$)

$$\begin{gathered} \dot R_1 R_1 = \dot R_2 R_2 \sim 4\nu, \quad \ddot R_i R_i \sim -16\nu^2 R_i^{-2}, \quad R_2 \sim \varepsilon_0 R_1^{-1}, \\ \varkappa = R_2/R_1 \sim \varepsilon_0 R_1^{-2}, \quad \dot\varkappa = \varkappa(\dot R_2/R_2 - \dot R_1/R_1) \sim -8\nu\varepsilon_0 R_1^{-4}; \end{gathered} \tag{3.11}$$

for the diverging ring ($R_1 \to R_{1*}$)

$$\begin{gathered} \dot R_i \sim 0, \quad \ddot R_i R_i \sim -\sigma_*, \quad R_2 \sim R_1 + \frac{\varepsilon_0}{R_1}, \\ \varkappa = R_2/R_1 \sim 1 + \varepsilon_0 R_1^{-2}, \quad \dot\varkappa \sim 0. \end{gathered} \tag{3.12}$$

4. Investigation of Stability. We investigate the stability of unsteady axisymmetric motion of a viscoplastic ring with free boundaries. We formulate the mathematical statement in Eulerian coordinates by analogy with [14]. We investigate the stability of the fundamental motion of the ring, determined by the equation of Sec. 3, for small perturbations of the velocity, the stress tensor, and the ring boundaries. Since the elementary perturbation is small, we assume that the principal direction in the perturbed motion corresponding to the direction of the tangent to the perturbed surface of the shell forms a small angle with the principal direction of the unperturbed ring.

The method for deriving the system of mathematical relations for perturbed motion of a strong ring with free boundaries is known [17, 18]. Hence, in view of the awkwardness of this type of problem, we indicate only the sequence of the calculations.

We express the components of the velocity, the stress tensor, and the ring boundaries in perturbed motion as a sum of the fundamental (axisymmetric) motion and small perturbations, and linearize Eqs. (2.1)-(2.5). The linearization of the incompressibility condition (2.2) of the medium under consideration ensures a sufficiently smooth flow function. Substituting the linearized components of the stress tensor (2.3) into the equations of motion (2.1), and expressing the velocity components in terms of the flow function, we obtain two linear equations for the average stress and the flow function. After differentiating one of these equations with respect to r and the other with respect to θ, and subtracting, we obtain for the flow function a linear partial differential equation for the fourth order in r and θ, and the first order in t.

The linearization of the kinematic (2.4) and the dynamic (2.5) boundary conditions, taking account of the mobility of the boundaries, gives six relations: four from (2.5) for the formulation of the boundary value problem for the flow function, and the other two for the determination of the perturbation of the shell boundaries for a known flow function and given initial conditions. The form of the differential equation for the flow function and the boundary conditions enables us to seek solutions for the average stress, the flow function, and the boundaries of the perturbed ring which are harmonic in θ. This substantially simplifies the problem, which after the introduction of the new variable $y = \ln r/R_1$ takes the very simple form

$$\begin{gathered} \frac{\partial}{\partial t}\left(\frac{\partial^2\psi}{\partial y^2} - \omega^2\psi\right) + \frac{\dot R_1}{R_1} e^{-2y}\left(\frac{\partial^3\psi}{\partial y^3} - 2\frac{\partial^2\psi}{\partial y^2} - \omega^2\frac{\partial\psi}{\partial y} + 2\omega^2\psi\right) \\ -\frac{\sigma_*}{4\dot R_1 R_1}\left(\frac{\partial^4\psi}{\partial y^4} - 2\omega^2\frac{\partial^2\psi}{\partial y^2} + \omega^4\psi\right) + \frac{\nu e^{-2y}}{2R_1^2}\left[\frac{\partial^4\psi}{\partial y^4} - 4\frac{\partial^3\psi}{\partial y^3} + (8-\omega^2)\frac{\partial^2\psi}{\partial y^2} + 4\omega^2\frac{\partial\psi}{\partial y} - 8\omega^2\psi\right] = 0 \quad (0 < y < \ln\varkappa); \end{gathered} \tag{4.1}$$

$$\frac{\partial^2\psi}{\partial t\,\partial y} + \frac{\dot R_i}{R_i}\frac{\partial^2\psi}{\partial y^2} - \frac{\sigma_*}{4\dot R_i R_i}\left[\frac{\partial^3\psi}{\partial y^3} + (\omega^2-4)\frac{\partial\psi}{\partial y} + 2\omega^2\psi\right] - \frac{\nu}{2R_i^2}\left(\frac{\partial^3\psi}{\partial y^3} - 2\frac{\partial^2\psi}{\partial y^2} - \omega^2\frac{\partial\psi}{\partial y}\right) = 0 \quad (y = \ln R_i/R_1); \tag{4.2}$$

$$\begin{gathered} \frac{\omega}{R_i}\left(\sigma_* + 4\nu\frac{\dot R_i}{R_i}\right)\xi_i + \left(\frac{\sigma_*}{4\dot R_i R_i} + \frac{\nu}{2R_i^2}\right)\left(\frac{\partial^2\psi}{\partial y^2} - 2\frac{\partial\psi}{\partial y} + \omega^2\psi\right) = 0 \\ (y = \ln R_i/R_1); \end{gathered} \tag{4.3}$$

$$\dot{\xi}_i - \frac{\omega}{R_i}\psi + \frac{\dot{R}_i}{R_i}\xi_i = 0 \quad (y = \ln R_i/R_1), \quad \xi_i(0) = \xi_{i0}, \quad \psi(0, y) = 0. \tag{4.4}$$

Here $\psi(t, y)$ and $\xi_i(t)$ are, respectively, the amplitudes of the harmonic perturbations of the flow function and the ring boundaries, ω is the wave number of the harmonic perturbation, $i = 1$ corresponds to the inner boundary of the ring, and $i = 2$ to the outer boundary.

The introduction of the new function $\varphi(t, y)$ by the formula

$$\varphi = \partial^2\psi/\partial y^2 - \omega^2\psi, \tag{4.5}$$

gives Eq. (4.1) the form

$$\frac{\partial\varphi}{\partial t} + \frac{\dot{R}_1}{R_1} e^{-2y}\left(\frac{\partial\varphi}{\partial y} - 2\varphi\right) - \frac{\sigma_*}{4\dot{R}_1 R_1}\left(\frac{\partial^2\varphi}{\partial y^2} - \omega^2\varphi\right) + \frac{\nu e^{-2y}}{2R_1^2}\left(\frac{\partial^2\varphi}{\partial y^2} - 4\frac{\partial\varphi}{\partial y} + 8\varphi\right) = 0. \tag{4.6}$$

5. Construction of the Solution for a Ring Converging toward Its Center. In this case the asymptotic relations (3.11) are valid. We introduce the small quantity $\beta = R_1^2 e^{2y}$ ($\beta \to 0$ as $R_1 \to 0$). Assuming $\dot{R}_1 R_1 \sim 4\nu$ in (4.6), we obtain the parabolic equation

$$\beta\frac{\partial\varphi}{\partial t} - \frac{\nu}{2}\left(\frac{\sigma_*\beta}{16\nu^2} - 1\right)\frac{\partial^2\varphi}{\partial y^2} + \frac{\sigma_*\omega^2\beta}{16\nu}\varphi = 0. \tag{5.1}$$

We note that Eq. (5.1) contains the small parameter β in a singular way [19, 20]. In this case the asymptotic expansion of the solution in terms of the small parameter takes the form

$$\varphi(t, y) = \sum_{n=0}^{\infty}\left[\varphi_1^n(t, y) + \varphi_2^n(\tau, y)\right]\beta^n, \quad \tau = t/\beta. \tag{5.2}$$

To shorten the calculations and to clarify the discussion in obtaining the asymptotic solution of the singularly perturbed parabolic equation (5.1), we assume β is constant. We note that the derivatives $\dot{\beta} = 2\beta\dot{R}_1/R_1$ and $\partial^n\beta/\partial y^n = 2n\beta$ do not introduce singularities as $\beta \to 0$, and affect only the order of accuracy in subsequent terms of the series with $n \geqslant 1$. For the first binomial of series (5.2) $\varphi^0 = \varphi_1^0 + \varphi_2^0$ we have from (5.1)

$$\frac{\partial^2\varphi_1^0}{\partial y^2} = 0, \quad \frac{\partial\varphi_2^0}{\partial\tau} + \frac{\nu}{2}\frac{\partial^2\varphi_2^0}{\partial y^2} = 0,$$

from which we obtain

$$\varphi_1^0 = c_1(t) + c_2(t)y, \quad \varphi_2^0 = \left(c_3 \operatorname{ch}\sqrt{\nu/2}\,y + c_4 \operatorname{sh}\sqrt{\nu/2}\,y\right)\exp(-\nu^2 t/4\beta), \tag{5.3}$$

where c_3 and c_4 are constants.

For a known φ^0 the solution for ψ^0 is determined with the same accuracy from (4.5) in the form

$$\psi^0 = c_5(t)e^{\omega y} + c_6(t)e^{-\omega y} + \frac{1}{\omega}\int_0^y \varphi^0(x)\operatorname{sh}\omega(y - x)\,dx. \tag{5.4}$$

Before substituting (5.4) into the boundary conditions (4.2) and (4.3) to obtain the system for determining the unknown functions of time, we list the values of integrals which will be needed later:

$$\int_0^\gamma \varphi^0(x)\operatorname{ch}\omega(\gamma - x)\,dx = \frac{c_1}{\omega}\operatorname{sh}\omega\gamma + \frac{c_2}{\omega^2}(\operatorname{ch}\omega\gamma - 1) + \frac{e^{-\frac{\nu^2 t}{4\beta}}}{\left(\frac{\nu}{2} - \omega^2\right)}(a_1c_3 + a_2c_4),$$

$$\int_0^\gamma \varphi^0(x)\operatorname{sh}\omega(\gamma - x)\,dx = \frac{c_1}{\omega}(\operatorname{ch}\omega\gamma - 1) + \frac{c_2}{\omega^2}(\operatorname{sh}\omega\gamma - \omega\gamma) + \frac{e^{-\frac{\nu^2 t}{4\beta}}}{\left(\frac{\nu}{2} - \omega^2\right)}(a_1c_4 + a_2c_3),$$

$$\int_0^\gamma \dot{\varphi}(x)\operatorname{ch}(\gamma - x)\,dx = \frac{\dot{c}_1}{\omega}\operatorname{sh}\omega\gamma + \frac{\dot{c}_2}{\omega^2}(\operatorname{ch}\omega\gamma - 1) - \frac{\nu^2\exp(-\nu^2 t/4\beta)}{4\beta\left(\frac{\nu}{2} - \omega^2\right)}(a_1c_3 + a_2c_4),$$

$$\gamma = \ln \varkappa, \quad a_1 = \sqrt{\frac{\nu}{2}} \operatorname{sh} \sqrt{\frac{\nu}{2}} \gamma - \omega \operatorname{sh} \omega\gamma,$$
$$a_2 = \sqrt{\frac{\nu}{2}} \operatorname{ch} \sqrt{\frac{\nu}{2}} \gamma - \omega \operatorname{ch} \omega\gamma, \quad \omega^2 \neq \nu/2. \tag{5.5}$$

For $\omega^2 = \nu/2$ the functions a_i take a somewhat different form. The third terms in (5.5) are of higher order in the small parameter β than the first two terms, and in constructing the asymptotic solutions they do not affect the first term of the asymptotic series.

Henceforth we replace the time derivative in terms of R_1 by the relation $d/dt = \dot{R}_1 d/dR_1$, and taking into account the asymptotic relations (3.11) and the values of the limits lim $\cosh(\omega \ln \varkappa)/\varkappa^{\omega} = 1/2$ and $\lim \tanh(\omega \ln \varkappa) = 1$ as $\varkappa \to \infty$, we obtain from (5.3)-(5.5) and (4.2), (4.3) the system

$$\begin{gathered}
4\omega\left(c_5' - c_6'\right) R_1 + 2\omega^2 (1 + 2\omega) c_5 - 2\omega^2 (2\omega - 1) c_6 - 10c_1 - c_2 = 0, \\
2\omega\left(c_5' - \varkappa^{-2\omega} c_6'\right) + \omega^{-2}\left(\omega c_1' + c_2'\right) - \frac{\sigma_* R_1}{(4\nu)^2}\Big[\omega(\omega^2 + \omega - 2) c_5 - \\
- \omega(\omega^2 - \omega - 2)\varkappa^{-2\omega} c_6 + \frac{(1+\omega)}{4\omega^2} c_1 + \frac{(\omega^{-2} + 2\omega^2 - 4)}{4\omega} c_2\Big] = 0, \\
2\omega(\omega - 1) c_5 + 2\omega(\omega + 1) c_6 + c_1 = -\omega q_1 \xi_1 / R_1, \\
2\omega(\omega - 1) c_5 + 2\omega(\omega + 1)\varkappa^{-2\omega} c_6 + \frac{1}{2}(1 - 2\omega^{-1} + \omega^{-2}) c_1 + \\
+ \frac{1}{2}(\omega^{-1} - 2\omega^{-2} + \omega^{-3}) c_2 = -\omega q_2 \frac{\xi_2}{R_1} \varkappa^{-\omega-1},
\end{gathered} \tag{5.6}$$

where $q_i = \left(\sigma_* + 4\nu \frac{\dot{R}_i}{R_i}\right) \Big/ \left(\frac{\sigma_*}{4\dot{R}_i R_i} + \frac{\nu}{2R_i^2}\right)$; $q_1 \sim 32\nu$; $q_2 \sim 16\nu$.

Taking account of the initial conditions for the amplitude of the perturbation of the flow function on the boundaries of the ring $\varphi^0(0, 0) = \varphi^0(0, \gamma)$ at $t = 0$, we can take $c_5(1) = c_6(1) = 0$ for $R_1 = 1$ when the constants c_j ($j = \overline{1, 4}$) are not equal to zero at $t = 0$. Hence, as $R_1 \to 0$ and $\omega \gg 1$, the solution of system (5.6) has the form

$$\begin{gathered}
c_1 = -2\omega\left(\omega I_1 + 16\nu\xi_1 R_1^{-1}\right), \quad c_2 = 2\omega^2\left(\omega I_1 + 16\nu\xi_1 R_1^{-1}\right), \\
c_5 = \frac{1}{2}(I_1 + I_2), \quad c_6 = \frac{1}{2}(I_1 - I_2), \\
I_1 = -8\omega\nu e^{-f_1} \int_1^{R_1} \left[p^{-2}\left(\xi_1 - \varkappa^{-\omega-1}\xi_2\right) + 4\omega^{-2} p^{-1} \varkappa^{-\omega-1} \xi_2'\right] e^{f_1(p)} dp, \\
I_2 = -8\omega\nu \int_1^{R_1} \left(\xi_1 - \varkappa^{-\omega-1}\xi_2\right) p^{-2} dp, \quad f_1(R_1) = \frac{a\omega^2}{2}\left(R_1^2 - 1\right), \quad a = \sigma_*/(4\nu)^2,
\end{gathered} \tag{5.7}$$

which is accurate to the leading terms.

Substituting (5.4) into (4.4) and replacing the time derivative in terms of the radius, we obtain for the amplitudes of the harmonic perturbation of the ring boundaries the differential equations

$$\begin{gathered}
\left(\xi_1' + R_1^{-1}\xi_1\right)\dot{R}_1 = \frac{\omega}{R_1}(c_5 + c_6), \quad \left(\xi_2' + \varkappa^{-1} R_2^{-1} \xi_2\right)\dot{R}_1 = \frac{\omega}{R_2}\left[c_5 \varkappa^{\omega} + c_6 \varkappa^{-\omega}\right. \\
\left. + \omega^{-2} c_1 (\operatorname{ch} \omega\gamma - 1) + \omega^{-3} c_2 (\operatorname{sh} \omega\gamma - \omega\gamma)\right].
\end{gathered} \tag{5.8}$$

We note that

$$c_5 \varkappa^{\omega} + c_6 \varkappa^{-\omega} = I_1 \operatorname{ch} \omega\gamma + I_2 \operatorname{sh} \omega\gamma,$$

and we introduce the new function

$$\eta = \varkappa^{-\omega-1}\xi_2, \quad \varkappa^{-\omega-1}\xi_2' = \eta' + (\omega + 1)\varkappa'\varkappa^{-1}\eta. \tag{5.9}$$

We derive a system of integrodifferential equations for determining ξ_1 and η from (5.7)-(5.9). In order to eliminate the two different kinds of integrals in the second equation of the system, we subtract twice the second equation from the first to obtain two first-order differential equations including I_1 or I_2. Isolating the integrals after differentiating with respect to the radius and using (3.11), we have from (5.7)-(5.9) a system of ordinary second-order differential equations

$$R_1^2\xi_1'' + (1 + a\omega^2 R_1^2) R_1\xi_1' + \omega^2(2 + aR_1^2)\xi_1 + 8R_1\eta' - 2\omega^2\eta = 0,$$
$$\varepsilon_0^2 R_1^2\eta'' - 2\omega\varepsilon_0^2 R_1\eta' + 10\omega\varepsilon_0^2\eta - \frac{1}{2}R_1^4(R_1^2\xi_1'' + R_1\xi_1' - 2\omega^2\xi_1) = 0. \quad (5.10)$$

The asymptotic expansion of the solution of the singularly perturbed system (5.10) in terms of the small parameter ε_0^2 is given by a series [20] analogous to (5.2). The first approximation to the solution of system (5.10) as $\varepsilon_0 \to 0$ and $R_1 \to 0$ has the form

$$\xi_1 \sim [\gamma_1(\alpha R_1^2)^m + \gamma_2(\alpha R_1^2)^{-m}]\exp\left(-\frac{1}{4}a\omega^2 R_1^2\right),$$
$$\eta \sim \gamma_3 R_1^{n_1} + \gamma_4 R_1^{n_2}, \quad n_{1,2} = \frac{1}{2}(1 + 2\omega \pm m_1), \quad (5.11)$$

where $m = i\omega/\sqrt{2}$; $\alpha = a\omega^2/2$; $m_1 = \sqrt{(1-2\omega)^2 - 32\omega}$; γ_j — const; $j = \overline{1, 4}$.

Considering the real part for ξ_1 and using (5.9), we obtain from (5.11) as $R_1 \to 0$ the asymptotic forms

$$\xi_1 \sim (\gamma_1 + \gamma_2)\cos\left(\frac{\omega}{\sqrt{2}}\ln \alpha R_1^2\right), \quad \xi_2 \sim \varepsilon_0^{\omega+1} R_1^{-3/2}(\gamma_3 + \gamma_4 R_1^{-2\omega}). \quad (5.12)$$

Consequently, in the inertial convergence of a viscoplastic ring toward the center, small perturbations of the boundaries increase without bound at the outer boundary, and on the inner boundary have a wavy character with a bounded amplitude. The perturbation on the outer boundary of the ring increases with increasing ε_0. This parameter characterizes the relative thickness of the ring wall at $t = 0$.

We note that in the inertial motion of a viscoplastic shell toward its axis the inside radius practically never reaches zero. For example [21], in experiments on the compression of cylindrical shells by a layer of explosives an explosive vaporization of inner layers of the shell was observed as the result of the transformation of kinetic energy into heat. There is an inner channel in the contracted shell. Denoting by R_{1*} the value of R_1 at the instant the ring stops moving, we have from (5.12), under the assumption of a maximum value of the amplitude of the harmonic perturbation on the inner boundary of the shell, $\alpha R_{1*}^2 = 1$. By using (3.1) we obtain the dependence of the wave number on the dimensional parameters in the form

$$\omega_0 = \frac{4\mu}{R_{1*}}\left(\frac{2}{\rho\sigma_s}\right)^{1/2}. \quad (5.13)$$

6. A Ring Expanding from the Center. In this case we have the asymptotic relations (3.12). We introduce the small parameter $\beta = \dot{R}_1/R_1$, and use (4.6) to obtain the singularly perturbed equation

$$4\beta R_1^2\frac{\partial\varphi}{\partial t} + 4\beta^2 R_1^2 e^{-2y}\left(\frac{\partial\varphi}{\partial y} - 2\varphi\right) - \sigma_*\left(\frac{\partial^2\varphi}{\partial y^2} - \omega^2\varphi\right) + 2\nu\beta e^{-2y}\left(\frac{\partial^2\varphi}{\partial y^2} - 4\frac{\partial\varphi}{\partial y} + 8\varphi\right) = 0. \quad (6.1)$$

The asymptotic expansion of the solution in terms of the parameter β has the form (5.2). We note the equality

$$\dot{\beta} = (\ddot{R}_1 R_1 - \dot{R}_1^2)R_1^{-2}, \quad \dot{\tau} = (1 - \dot{\beta})\beta^{-1}, \quad \text{где } \tau = t/\beta. \quad (6.2)$$

To first-order terms as $\beta \to 0$ we have

$$\frac{\partial^2\varphi_1^0}{\partial y^2} - \omega^2\varphi_1^0 = 0, \quad 4R_1^2(1 - \dot{\beta})\frac{\partial\varphi_2^0}{\partial\tau} - \sigma_*\left(\frac{\partial^2\varphi_2^0}{\partial y^2} - \omega^2\varphi_2^0\right) = 0. \quad (6.3)$$

We obtain from (6.2) and (6.3)

$$\varphi_1^0(t, y) = C_1(t)\,\mathrm{ch}\,\omega y + C_2(t)\,\mathrm{sh}\,\omega y,$$
$$\varphi_2^0(t, y) = (C_3 + C_4 y)\exp\left[-\omega^2\int_0^{t/\beta} b(\tau)\,d\tau\right], \quad (6.4)$$

where $b = \sigma_*/4R_1^2(1 - \dot{\beta})$ $(C_3, C_4$ — const).

For a known function $\varphi^0 = \varphi_1^0 + \varphi_2^0$ we determine from (4.5) the value for ψ^0 by using Eq. (5.4). Repeating the series of arguments of Sec. 5, noting the limiting values (3.12), in particular $\sinh\gamma\varkappa \sim \varepsilon_0 R_1^{-2}$ and $\cosh\gamma\varkappa \sim 1$, and considering the solution of the system for the unknown functions of time in the form of an asymptotic series in the small parameter [20], we obtain for the first term of the series (for $\varepsilon_1 = 0$, when $\xi_1 = \xi_2 = \xi$) the following equation for ξ:

$$\frac{d\xi}{d\tau}\dot{\tau} + \left(\beta + \frac{q_1}{2R_1^2}\right)\xi + bR_1^{-1}\left[C_2 + 3\omega^2 e^{-f_2}\int_0^\tau bC_2 e^{f_2(\tau)}d\tau\right] = 0, \tag{6.5}$$

where q_1, f_2, and C_2 as functions of time are determined from the relations

$$\begin{gathered} q_1 = 4\beta R_1^2(\sigma_* + 4\nu\beta)/(\sigma_* + 2\nu\beta), \quad f_2(\tau) = -3\omega^2\int_0^\tau b(\tau)\,d\tau, \\ \frac{d^2C_2}{d\tau^2} + (2 - 3\omega^2 b)\frac{dC_2}{d\tau} - 2b(2 + 3\omega^2)C_2 = 0. \end{gathered} \tag{6.6}$$

Differentiating Eq. (6.5) with respect to τ and isolating the integral, we obtain

$$\begin{gathered} B\frac{d^2\xi}{d\tau^2} + \left(\frac{dB}{d\tau} + A - 3\omega^2 b\right)\frac{d\xi}{d\tau} + \left(\frac{dA}{d\tau} - 3\omega^2 bA\right)\xi + \frac{dC_2}{d\tau} = 0, \\ B = \frac{4R_1^3(1-\dot{\beta})^2}{\beta\sigma_*}, \quad A = \frac{B}{\dot{\tau}}\left(1 + 2\frac{\sigma_* + 4\nu\beta}{\sigma_* + 2\nu\beta}\right). \end{gathered} \tag{6.7}$$

We introduce the new variable $p = R_1/R_{1*} - 1$, where R_{1*} is the value of the inside radius of the ring at the instant it stops moving. For the expansion of the ring we have $R_1 \to R_{1*}$, from which $p \to 0$. Transforming to the new variable in (6.6) and (6.7), we obtain a system of ordinary second-order differential equations for $\xi(p)$ and $C_2(p)$

$$\begin{gathered} B\left(\frac{dp}{d\tau}\right)^2\xi'' + \left[B\frac{d^2p}{d\tau^2} + \left(\frac{dB}{d\tau} - 3\omega^2 b + A\right)\frac{dp}{d\tau}\right]\xi' + \left(\frac{dA}{d\tau} - 3\omega^2 bA\right)\xi + C_2'\frac{dp}{d\tau} = 0, \\ \left(\frac{dp}{d\tau}\right)^2 C_2'' + \left[\frac{d^2p}{d\tau^2} + (2 - 3\omega^2 b)\frac{dp}{d\tau}\right]C_2' - 2b(2 + 3\omega^2)C_2 = 0, \end{gathered} \tag{6.8}$$

where primes denote differentiation with respect to p.

It follows from (3.12) with an accuracy up to first-order terms in p that $\beta \sim \dot{p}(1-p)$, $\dot{\beta} \sim -[\sigma_* R_{1*}^{-2} + \dot{p}^2](1-2p)$, $\ddot{\beta} \sim 2\dot{p}(2\sigma_* R_{1*}^{-2} + \dot{p}^2)(1-3p)$, from which

$$\frac{dp}{d\tau} = \dot{\tau}^{-1}\dot{p} \simeq \dot{p}^2(1+k)^{-1}, \quad \frac{d^2p}{d\tau^2} = \dot{\tau}^{-1}\frac{d}{d\tau}\left(\frac{\dot{p}}{\dot{\tau}}\right) \simeq -2k(1+k)^{-2}\dot{p}^2, \quad k = \sigma_* R_{1*}^{-2}.$$

After linearizing the coefficients in (6.8), we obtain the system

$$\begin{gathered} \dot{p}^3\xi'' - k\dot{p}\xi' - \frac{9\omega^2}{4}k\xi + \frac{k\dot{p}^2}{4R_{1*}(1+k)}C_2' = 0, \\ \dot{p}^4 C_2'' + \left(2 - \frac{3\omega^2 k}{4}\right)\dot{p}^2 C_2' - \left(1 + \frac{3}{2}\omega^2\right)k(1+k)C_2 = 0. \end{gathered} \tag{6.9}$$

For a fixed value of the small parameter $\dot{p}$ the solution of system (6.9) has the form [16]

$$\begin{gathered} \xi = m_1\exp(n_1 p/\dot{p}^2) + m_2\exp(n_2 p/\dot{p}^2) - \frac{k\dot{p}R_{1*}^{-1}}{2\lambda(k+1)}\int_0^p C_2'(q)\, e^{-\frac{k}{2\dot{p}^2}(q-P)}\operatorname{sh}\frac{\lambda(p-q)}{2\dot{p}^2}\,dq, \\ C_2 = m_3\exp(n_3 p/\dot{p}^2) + m_4\exp(n_4 p/\dot{p}^2). \end{gathered} \tag{6.10}$$

Here the m_j, where $j = \overline{1,4}$, are integration constants, and the n_j and λ are given by the equations

$$\begin{gathered} n_1 = \frac{1}{2}(\lambda + k), \quad n_2 = -\frac{1}{2}(\lambda - k), \quad \lambda = k\sqrt{1 + 9\omega^2\dot{p}/k}, \\ n_{3,4} = \pm\lambda_1 - 1 + \frac{3}{8}k\omega^2, \quad \lambda_1 = \left[1 + k\left(1 + k + \frac{3}{4}\omega^2 + \frac{3}{2}\omega^2 k\right)\right]^{1/2}. \end{gathered}$$

Taking account of the limiting equations

$$\lim_{p\to 0,\dot{p}\to 0} (p/\dot{p}) = 0, \quad \lim_{p\to 0,\dot{p}\to 0} (p/\dot{p}^2) = 1/2k,$$

we have from (6.10) in the asymptotic approximation the following relation for the amplitude of a harmonic perturbation of the boundaries of the ring:

$$\xi \sim \text{const} + \text{const}\left[\dot{p}\left(n_3 - k - \frac{9}{4}\omega^2\dot{p}\right)\right]^{-1},$$

from which it follows that $\xi \to \infty$ as $\dot{p} \to 0$. In the special case $n_3 = k$, the instability of the motion of the ring is extreme. For a rapid expansion of the shell we have from $n_3 = k$ a relation for the wave number $\omega_*^2 \simeq (8/3k)(1 + 5k/4)$. Let $\varepsilon_* = \ln R_{1*}/R_{10}$ be the limiting value of the logarithmic strain of the ring. Using (3.1), we obtain the following relation for the wave number characterizing the unstable deformation of a viscoplastic shell as a function of the dimensional parameters:

$$\omega_* = \frac{2V_{10}e^{\varepsilon_*}}{\sqrt{3\sigma_s/2\rho}}\left(1 + \frac{5\sigma_s}{4\rho V_{10}^2}e^{-2\varepsilon_*}\right)^{1/2}. \tag{6.11}$$

7. Discussion. Let us consider the experimental evaluation of Eqs. (5.13) and (6.11). There are experimental data [22] on the impulsive loading of metal tubes by the explosion of charges located concentrically on the outer surface of a shell. Seamless tubes of St 10 and St 20 were used in the experiments. The experiments show that the convergence of the tube walls toward its axis is accompanied by buckling and the formation of waves on the shell boundaries [22, 23].

Table 1 lists experimental data on the specific impulse I_0, the strain $\varepsilon_{2*} = (R_{20} - R_{2*})/R_{20}$ in the central cross section of the compressed tube, and the number of waves (wrinkles) ω_e in this cross section. In experiments 1-3 the tube material was St 10, and in the remainder it was St 20. The dimensions of the tubes $R_{20} \times s_0 \times h$ in experiments 1, 2, 4-6 were 54 × 4 × 216 mm; in experiment 3 they were 54 × 4 × 1080 mm; in experiment 7 they were 54 × 4 × 360 mm. The number of waves ω_0 was calculated with (5.13), using a dynamic viscosity for steel [24, 25] $\mu = (4\text{-}5)\cdot 10^4$ kg/(m·sec) and a density $\rho = 7.85\cdot 10^3$ kg/m^3. The dynamic yield point for St 10 and St 20 was calculated from statistical experimental data according to [26], using $\sigma_s = 0.37$ GPa and $\sigma_s = 0.41$ GPa, respectively. The value of R_{1*} was determined from the incompressibility condition of the material, from which it follows that

$$R_{2*}^2 - R_{1*}^2 = R_{20}^2 - R_{10}^2, \text{ where } R_{2*} = R_{20}(1 - \varepsilon_{2*}).$$

The well-known computational formulas [5, 27] from the dynamical theory of the buckling of cylindrical shells under plastic flow give a rougher estimate than (5.13) or the formula in [28] derived for the elastic buckling of thin-walled tubes. This problem was treated in more detail in [2, 22, 28]. We note that for thin-walled shells ($s_0/R_{20} \leqslant 0.05$) the calculation of the harmonics with Eq. (5.13) underestimates the experimental result. The introduction of the factor $\sqrt{R_{20}/s_0}$ into (5.13) gives good agreement with experiment also in the case of intrinsically thin-walled shells (cf. experimental data in [4, 28]).

We consider the experimental result on the reduction of a tube of St 20 without a mandrel (Fig. 3). Here with $R_{1*} = 26.5$ mm and the remaining parameters as listed above, we obtain from (5.13) $\omega_0 \simeq 4$, which agrees with experiment. Laboratory and industrial research showed that the difference in transverse wall thickness was less for the longitudinal rolling of tubes on multistand mills than when rollers with a wavy profile were used in the first stands of the mill ($\omega_0 = 6\text{-}8$) [29].

We now discuss Eq. (6.11) for estimating fragment formation in the fracture of metal rings and tubes by detonation products. Theoretical [12, 17] and experimental [30, 31] results show that the number of fragments varies nearly linearly with the initial velocity of expansion of the shell. Table 2 shows experimental data [30] on the number of fragments ω_e in the fracture of aluminum rings with $\sigma_s = 0.12$ GPa, $\varepsilon_* = 0.28$, and $\rho = 2.75\cdot 10^3$ kg/m^3. The thickness s_0 of the ring wall and the velocity of expansion were varied. The ring and the explosive charge were 10 mm high, and the inside radius was 20 mm. We introduce into (6.11) the scale factor $(R_{20}/s_0)^{3/2}$ by the formula $\omega_*' = \omega_*(R_{20}/s_0)^{2/3}$. The calculated ($\omega_*'$) and experimental ($\omega_e$) values in Table 2 show satisfactory agreement.

TABLE 1

No.	$I_0 \cdot 10^{-2}$ kN·sec/m²	ε_{2*}	ω_e	ω_0
1	0,10	0,11	6	3—4
2	0,10	0,36	6	5—6
3	0,10	0,41	8	5—7
4	0,23	0,36	8	5—6
5	0,63	0,77	7	8—10
6	0,53	0,53	7	8—10
7	0,16	0,72	7	9—11

TABLE 2

No.	V_{10}, m/sec	s_0, mm	ω_e	ω'_*	ω'_1
1	450	2	20	24	28
2	240	4	12	10	8
3	200	6	6	7	5
4	1000	2	40	50	63
5	500	4	23	18	17
6	370	6	16	11	9
7	1100	4	50	37	38

Using the well-known fracture criterion, we derived in [32] the expression $l_* = c\varepsilon_{2*}R_{20}/V_{10}$ for the average length of a fragment in the fracture of a metal shell expanding inertially. Here c is the speed of sound in the ring material, and $\varepsilon_{2*} = R_{2*}/R_{20} - 1$. Hence [12] the number of fragments is determined by the equation $\omega_1 = 2\pi R_{2*}/l_*$. By taking R_{20}/s_0 as a scale factor, we obtain

$$\omega'_1 = \frac{2\pi V_{10} R_{20}}{c s_0 \varepsilon_{2*}} (1 + \varepsilon_{2*}). \tag{7.1}$$

The results calculated with (7.1) for the experiments [30] described above and summarized in Table 2 show that ω'_* and ω'_1 are close to one another. We note that the accuracy of the quantitative comparison of the calculated and experimental values depends strongly on the magnitude of the limiting (logarithmic) strain. In general the experimental values ω_e depend on the strain rate and the sample thickness, whereas ω was assumed constant in the above calculations. This may account for the necessity of introducing scale factors of the type R_{20}/s_0.

Thus, our study of the problem of the stability of inertial motion of a viscoplastic cylindrical shell under small harmonic perturbations of the boundaries, the velocity, and the stress tensor enables us to determine the wave number of the harmonic for which the maximum increase of the amplitude of the perturbation of the boundaries is expected. In the contraction of a ring toward its axis, the number of unstable harmonics depends strongly on the dynamic viscosity of the shell material, whereas in the expansion of a ring this dependence is of second order. The relations derived for the number of wrinkles and fragments in the buckling (fracture) of actual metal rings are not inconsistent with the experimental and theoretical results, and in some cases agree well with experiment.

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